State reconstruction for composite systems of two spatial qubits

G. Lima*,[1] F.A. Torres-Ruiz,[2] Leonardo Neves †,[1] A. Delgado,[2] C. Saavedra,[2] and S. Pádua[1]

[1]Departamento de Física, Universidade Federal de Minas Gerais,
Caixa Postal 702, Belo Horizonte, MG 30123-970, Brazil.‡
[2]Center for Quantum Optics and Quantum Information, Departamento de Física,
Universidad de Concepción, Casilla 160-C, Concepción, Chile.

Pure entangled states of two spatial qudits have been produced by using the momentum transverse correlation of the parametric down-converted photons [Phys. Rev. Lett. **94** 100501]. Here we show a generalization of this process to enable the creation of mixed states of spatial qudits and by using the technique proposed we generate mixed states of spatial qubits. We also report how the process of quantum tomography is experimentally implemented to characterize these states. This tomographic reconstruction is based on the free evolution of spatial qubits, coincidence detection and a filtering process. The reconstruction method can be generalized for the case of two spatial qudits.

I. INTRODUCTION

The concept of quantum state plays a central role in the Quantum Theory. It is considered to be the most complete description for a physical system. Statistical distributions of results of experiments carried out on a physical system can be completely predicted from its initial state. Therefore, the experimental determination of an initially unknown quantum state becomes a very important research subject. This has led to the development of techniques to perform the state determination. In recent years the problem of state determination has received a considerable degree of attention due to important results in Quantum Information Theory.

Several techniques have been designed and used for the state estimation of different physical systems. In the field of atomic physics, quantum endoscopy was used to determine the state of ions and atoms [1, 2, 3]. In quantum optics, the Wigner function of multi mode fields could be measured using homodyne detection [4, 5, 6] and the technique of quantum tomographic reconstruction (QTR) was used for measuring the polarization state of parametric down-converted photons [7].

In general, these methods are based on a linear inversion of the measured data. In the case of QTR, the data is acquired with a series of measurements performed on a large number of identically prepared copies of a quantum system. The fact that this transformation is linear, makes it strongly dependent of any experimental error that may occur while recording the data. It can appear as a consequence of the experimental noise or misalignment and therefore, the reconstructed state is only a reasonable approximation of the real quantum state. The density

matrices obtained may have properties that are not fully compatible with a quantum state. Another alternative that has been considered for the state determination is the numerical technique called maximum likelihood estimation [8, 9]. It is based on a relation between the measured data and the quantum state that could have generated them. Even though it generates only possible density matrices, it has the drawback of enhancing the uncertainty on the state estimation.

In this article we are interested in the determination of the state of a composite system. In our experiment, the system corresponds to two photons generated by spontaneous parametric down-conversion (SPDC). In this non-linear process a photon from a pump laser beam incident to a non-linear crystal originates probabilistically two photons, signal and idler [10]. The photons of this pair are also called twin photons for being generated simultaneously [11]. Recently, we have demonstrated that by placing D symmetric slits at the path of each twin photon it is possible to use the transverse correlations of the photon pair to generate maximally entangled states of two effective D-dimensional quantum systems [12, 13, 14]. We refer to these D-dimensional quantum systems as spatial qudits. In the present work we extend previous results to the generation of mixed states of two spatial qubits, which also applies to the case of qudits. Following this, we investigate the state determination of two spatial qubits. We show both theoretically as well as experimentally, that one can implement the process of QTR to obtain the density operator of a state composed of two spatial qubits. The quality of the reconstruction performed is also discussed. Even though we had considered only the special case of spatial qubits, it is straightforward to show that the technique used can be generalized for being applied to a system composed of two spatial qudits. The main motivation on studying both the generation and the reconstruction of mixed states of spatial qudits is to consider more realistic experimental situations in case of using them in technological fields, such as quantum communication, where pure states can become into mixed ones due to interactions with their environment.

*Present address: Group of Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland.

†Present address: Clarendon Laboratory, University of Oxford, Parks Road Oxford Ox 13 PU, UK.

‡Electronic address: glima@cfm.cl

II. CONTROLLED GENERATION OF MIXED STATES

It was shown in [12, 13] that the state of parametric down-converted photons when each photon is transmitted through identical multi slits is given by

$$| \Psi \rangle = \sum_{l=-l_D}^{l_D} \sum_{m=-l_D}^{l_D} W_{lm} \, \exp\left(i\frac{kd^2}{8z_A}(m-l)^2 \right) | l \rangle_s \otimes | m \rangle_i,$$

(1)

where D is the number of slits in each multi slits array, d is the distance between the center of two consecutive slits, a is the half width of the slits and $l_D = (D-1)/2$. The function W_{lm} is the spatial distribution of the pump beam at the plane of the multi slits ($z = z_A$) and at the transverse position $x = (l+m)d/2$,

$$W_{lm} = W\left[\frac{(l+m)d}{2}; z_A\right].$$

(2)

The $| l \rangle_s$ (or $| m \rangle_i$) state is a single-photon state defined, up to a global phase factor, by the expression

$$| l \rangle_j \equiv \sqrt{\frac{a}{\pi}} \int dq_j \, \exp(-iq_j ld) \, \mathrm{sinc}\,(q_j a)| 1q_j \rangle,$$

(3)

and represents the photon in mode j ($j = i, s$) transmitted by the slit l. The transverse component of wave vector of the down-converted photons in the mode j is represented by q_j. The states in the set $\{| l \rangle_j\}$ are orthonormal, that is $_j\langle l | l' \rangle_j = \delta_{ll'}$. We use these states to define the logical states of the qudits. In this sense (1) describes a composite system of two qudits. Each qudit is represented by a state in a Hilbert space of dimension D, being D the number of available paths for its transmission through the multi slits array.

It can be seen from (1) and (2), that it is possible to create different pure states of spatial qudits if one knows how to manipulate the pump beam in order to generate distinct transverse profiles at the plane of the multi slits ($W(\xi; z_A)$) [13]. Let us now assume that, before reaching the crystal, the pump beam pass through an unbalanced Mach-Zehnder interferometer where the transverse profile of the laser beam is modified differently in each arm. If the difference between the lengths of these arms is set larger than the laser coherence length, we will obtain an incoherent superposition of the spatial qudits states generated by each arm.

We show in the following section how to use the QTR technique to determine the density matrix of these composite systems. The state whose the density matrix is reconstructed experimentally is a mixed state of two spatial qubits. This state is generated with the experimental setup represented in figure 1(a). A 5 mm β-barium borate crystal is pumped by a 500 mW krypton laser emitting at $\lambda = 413$ nm for generating SPDC. Before being incident at the crystal, the pump beam cross an unbalanced Mach-Zehnder interferometer. The difference between the lengths of each interferometer arm (200 mm) is

set larger than the laser coherence length (80 mm). Two identical double slits A_s and A_i are aligned in the direction of the signal and idler beams, respectively, at a distance of 200 mm from the crystal (z_A). The slits' width is $2a = 0.09$ mm and their separation, $d = 0.18$ mm. The smaller dimension of the double-slits are in the x-direction. All measurements are done in the x-axes, at the detection plane. At the arm 1 of the interferometer, we place a lens that focus the laser beam at the plane of these double slits, into a region smaller than d. In arm 2, we use a set of lenses that increases the transverse width of the laser beam at z_A. The transverse profiles generated are illustrated in figure 1(a). The photons transmitted through the double-slits are detected in coincidence between the detectors D_i and D_s. Two identical single slits of dimension 5.0 x 0.1 mm and two interference filters centered at 826 nm with 8 nm full width at half maximum (FWHM) bandwidth are placed in front of the detectors.
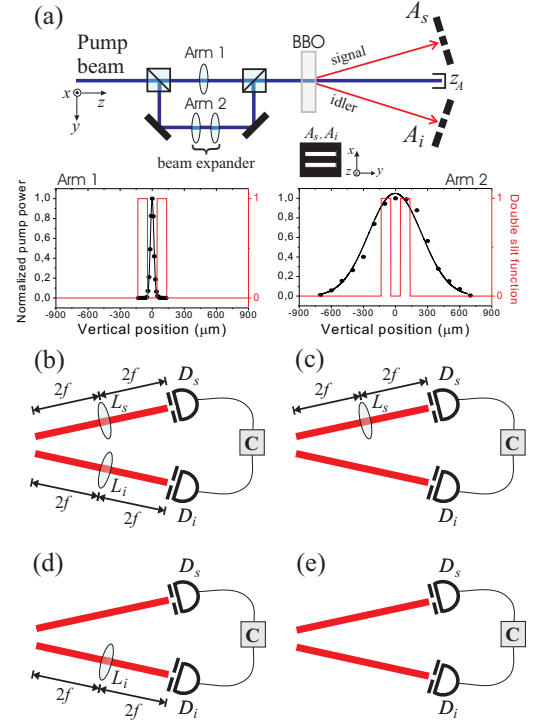

FIG. 1: (a) Schematic diagram of the experimental setup used for generating and characterizing the mixed states of spatial qubits. The pump beam that cross arm 1 is focused in a narrow region at z_A or in a broader spatial region when it cross arm 2. A_s and A_i are the double-slits at signal and idler propagation paths, respectively. D_s and D_i are detectors and C is a photon coincidence counter. The configuration used to determine the diagonal elements is represent in (b). (c) and (d) were used for the second type of measurement and (e) for the third type. All measurements are done in the x-axes, at the detection plane.

By using (1) and (2), we can show that the two-photon state, after the double slits, when only arm 1 is open is

given by

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2}}(|+\rangle_s|-\rangle_i + |-\rangle_s|+\rangle_i). \qquad (4)$$

To simplify, we used the state $|+\rangle_j$ and $|-\rangle_j$ in place of the states $|\frac{1}{2}\rangle_j$ and $|\frac{-1}{2}\rangle_j$. Thus, the state $|+\rangle_j$ $(|-\rangle_j)$ represents the photon in mode j being transmitted by the upper (lower) slit of its double slit. The state of (4) is a maximally entangled state of two spatial qubits and their correlation is such that when the idler photon passes through the upper (lower) slit of its double slit, the signal photon will only pass through its lower (upper) slit, and vice-versa.

However, if the laser beam cross only arm 2, the state of the twin photons transmitted by these apertures will be given by

$$
\begin{aligned}
|\Psi\rangle_2 &= \frac{1}{2}e^{i\phi}(|-\rangle_s|+\rangle_i + |+\rangle_s|-\rangle_i) \\
&\quad + \frac{1}{2}(|-\rangle_s|-\rangle_i + |+\rangle_s|+\rangle_i),
\end{aligned} \qquad (5)
$$

where $\phi = kd^2/8z_A$. And now the correlation of this spatial qubits is different since we can also have both photons of the pair generated, crossing their upper or lower slits, simultaneously.

Therefore, the two-photon state generated in our experiment when the two arms are liberated, is a mixed state of the spatial maximally entangled state shown in (4) and the state of (5). It is described by the density operator

$$\rho_{\text{the}} = A|\Psi\rangle_{11}\langle\Psi| + B|\Psi\rangle_{22}\langle\Psi|. \qquad (6)$$

where A and B are the probabilities for generating the states of arm 1 and arm 2, respectively.

III. RECONSTRUCTION

Now, we show how QTR can be experimentally implemented to reconstruct the density operator of the state (6) generated in our setup without the use of any information about the scheme used for this generation.

Let us briefly review the process of quantum tomography. The diagonal elements of any density operator can be measured directly. Therefore, quantum tomographic reconstruction is a protocol to determine the non-diagonal elements. It consists in the use of known unitary transformations on the system. Each transformation generates a new density operator whose diagonal elements are a combination of the coefficients of the transformation and of the non-diagonal elements of the original density operator. These new diagonal elements can be measured. The iteration of this procedure creates a set of equations which allows the determination of the non-diagonal elements of the initially unknown density operator. For a detailed account on this subject we refer the reader to [15, 16].

To characterize the density operator generated in our experiment, we first adopt a general form for it

$$\rho = \sum_{l,m=\pm} \rho_{l_s l_i m_s m_i}|l_s, l_i\rangle\langle m_s, m_i|. \qquad (7)$$

As we are dealing with two qubits states, the total number of measurement basis necessary for the QTR is nine [17]. They can be generated by using three basis for each qubit. Since we have spin $1/2$ like systems, these basis are the eigenvectors of the Pauli operators $\{\sigma_x, \sigma_y, \sigma_z\}$. In our case, the eigenvectors of σ_z are the slit's states given by (3). They form the logical base, such that $U_j^{(z)} = \mathbb{I}_j$. Measurements in basis $\{\sigma_x, \sigma_y\}$ in mode j require local unitary operations $\{U_j^{(x)}, U_j^{(y)}\}$, which allow us for going from σ_z eigenvectors to σ_x, σ_y eigenvectors, respectively. The density operator under these local transformations can be written as

$$\rho^{(\lambda,\mu)} = U_s^{(\lambda)} \otimes U_i^{(\mu)} \rho U_s^{(\lambda)\dagger} \otimes U_\mu^{(y)\dagger}, \qquad (8)$$

with $\lambda, \mu = x, y$. Since the σ_z eigenvectors are the slit's states, the question which remains is: How one can implement the above discrete local operations in these slit's states to perform the QTR of ρ? For answering this, we first consider the state of a photon crossing a given slit l $(l = \pm)$ along mode j $(j = s, i)$, and propagating through the free space to a detection plane located at position z. This state can be calculated by the method presented in [20] and is described by

$$|g_l\rangle_j = \sqrt{\frac{a}{\pi}} \int dq_j \, \exp(-i\alpha q_j^2)\exp(-ilq_jd)\,\text{sinc}\,(q_ja)|1q_j\rangle. \qquad (9)$$

It corresponds to the free evolution of the state $|l\rangle_j$ generated by the unitary operator U_j (restricted to the one-photon subspace), that is $|g_l\rangle_j = U_j|l\rangle_j$. The operator U_j is given by

$$U_j = \exp(-ik(z - z_A)) \int dq \, \exp(-i\alpha q^2)|1q\rangle_{jj}\langle 1q|, \qquad (10)$$

where $\alpha = (z - z_A)/2k$.

Therefore, if we propagate the state ρ from the plane-z_A to plane-z the result is

$$\rho_z = \sum_{l,m=+,-} \rho_{l_s l_i m_s m_i}|g_{l_s}, g_{l_i}\rangle\langle g_{m_s}, g_{m_i}|, \qquad (11)$$

and it becomes clear that ρ and ρ_z have the same coefficients, and thus, that one can reconstruct ρ by determining ρ_z, i.e, by doing the measurements in the detection plane-z.

It can be deduced from (9) that photons spread out along the measurement plane, so that we have passed

from discrete variables, states $|\,l\,\rangle_j$, to a continuously distributed state $|\,g_l\,\rangle_j$. Thus, for carrying out the measurement in discrete basis in plane-z, we need to implement an adequate postelection process, which will allow us to recover the discrete nature of the logical states. As we shall show in the following lines, this can be properly done by allocating at the detection plane two new slits for each mode j.

In order to explicitly show how the transformations $U_j^{(\lambda)}$ and $U_j^{(\mu)}$ $(\lambda, \mu = x, y)$ can be implemented to do the QTR for ρ_z, we first write an arbitrary two photon pure state in the transverse plane-z as

$$|\,\Psi\,\rangle_z \propto \sum_{m,n=\pm} A_{m,n}|\,g_m\,\rangle_s|\,g_n\,\rangle_i \qquad (12)$$

The transmitted state through the double slits placed at the detection plane is

$$|\,\Psi_{T_{\lambda\mu}}\,\rangle_z = \int dq_s \int dq_i\, F_{T_{\lambda\mu}}(q_s,q_i)|\,1_{q_s}\,\rangle|\,1_{q_i}\,\rangle, \qquad (13)$$

where the transmitted state biphoton amplitude is given by [20]

$$F_{T_{\lambda\mu}}(q_s,q_i) = \int dq_s' \int dq_i'\, F(q_s',q_i')T_\lambda(q_s'-q_s)T_\mu(q_i'-q_i). \qquad (14)$$

$T(q_j)$ is the Fourier transform of the double slits transmission function at mode j

$$T_\mu(q_j) \propto \left(e^{iq_j x_{\mu,0}} + e^{iq_j x_{\mu,1}}\right)\operatorname{sinc}\left(q_j b\right), \qquad (15)$$

where $x_{\mu,k}$ is the position of the slit k $(k=0,1)$ in mode j at the plane-z. By replacing the expressions for the states $|\,g_l\,\rangle_j$ in $|\,\Psi\,\rangle_z$, we determine the biphoton amplitude $F(q_s,q_i)$ in (12). By inserting $F(q_s,q_i)$ and (15) into (14), we obtain $F_{T_{\lambda\mu}}(q_s,q_i)$. After a straightforward derivation, we can rewrite the two photon transmitted state (13) as

$$|\,\Psi_{T_{\lambda,\mu}}\,\rangle_z \propto \sum_{k,l=0,1} B_{k,l}|f(x_{\lambda,k})\rangle_s|f(x_{\mu,l})\rangle_i, \quad (16)$$

where

$$B_{k,l} = \sum_{m,n=\pm} r_m(x_{\lambda,k})r_n(x_{\mu,l})A_{m,n}. \qquad (17)$$

The states of the post-selected photons which crossed this additional pair of slits are described by

$$|f(x_{\mu,k})\,\rangle_j \equiv \sqrt{\frac{b}{\pi}} \int dq_j'\, \exp(-iq_j' x_{\mu,k})\operatorname{sinc}\left(q_j'\frac{x_{\mu k}}{2\alpha} + q_j' b\right)|1q_j'\,\rangle, \qquad (18)$$

where $x_{\mu,k}$ is the position of the slit k $(k=0,1)$ in mode j at the plane-z. The transversal position of these slits determines which effective unitary operation was performed at the transmitted photon. We have also defined

$$r_\pm(x_{\mu,k}) = \exp\left(i\frac{(x_{\mu,k}\mp d)^2}{4\alpha}\right)\operatorname{sinc}\left(\frac{(x_{\mu,k}\mp d)}{2\alpha}\right). \qquad (19)$$

Here, by comparing (16) and (12), it can be observed that the post selection process acts on the $|\,g_\pm\,\rangle_j$ states with the following effective transformation

$$U_j^{(\mu)}|\,\tilde{g}_\pm\,\rangle_j \propto r_\pm(x_{\mu,0})|f(x_{\mu,0})\rangle_j + r_\pm(x_{\mu,1})|f(x_{\mu,1})\rangle_j, \qquad (20)$$

where $|\,\tilde{g}_\pm\,\rangle_j$ state denotes the post selected state arising from $|\,g_\pm\,\rangle_j$ state. By considering the value of the experimental parameters: $z - z_A$, d, a and b, it can be shown that the states $|f(x_{\mu,0})\rangle_j$ and $|f(x_{\mu,1})\rangle_j$ are orthogonal when the condition $|x_{\mu,1} - x_{\mu,0}| > 4b$ is satisfied.

The positions of the new slits for generating the effective transformation $U_j^{(x)}$ $(U_j^{(y)})$ are $x_{x,0} = 0$ $(x_{y,0} = -\Delta/2)$ and $x_{x,1} = \Delta$ $(x_{y,1} = \Delta/2)$, with $\Delta = \frac{\alpha\pi}{d} = 1376$ mm (Note that condition $|x_{\mu,1} - x_{\mu,0}| = \Delta > 4b$ is widely satisfied). We remark that we refer to the transformations done by these slits as effective transformations, due to the fact that they act as unitary transformations only for the photons in mode j which were transmitted through the pair of slits at the detection plane.

Because of the linearity of quantum mechanics and because we are performing only local operations to the twin photons, we know that the diagonal elements of $\rho^{(\lambda,\mu)}$ operators are linear combinations of the coefficients $\rho_{l_s l_i m_s m_i}$ of (11). The diagonal elements of the transformed density operators $\rho^{(\lambda,\mu)}$ at the plane-z are simply determined by using four coincidence numbers measured when D_s is at the position $x_{\lambda,k}$ for $k = 0, 1$ and D_i is fixed at the transversal position $x_{\mu,0}$ or at $x_{\mu,1}$. We assume that detectors D_s and D_i are placed just behind

the new slits at the plane-z. These diagonal elements are then

$$\rho_{L_s L_i, L_s L_i}^{(\lambda,\mu)} = \sum_{l_s l_i m_s m_i = \pm} U_{L_s,l_s}^{(\lambda)} U_{L_i,l_i}^{(\mu)} \rho_{l_s l_i m_s m_i} U_{m_s,L_s}^{(\lambda)} U_{m_i,L_i}^{(\mu)},$$

$$(21)$$

where $L_s, L_i = 0,1$ for $\lambda, \mu = x,y$ or $L_s, L_i = \pm$ for $\lambda, \mu = z$, and $U_{L_j,l_j}^{(\mu)}$ are coefficients of the effective transformation $U_j^{(\mu)}$ given by (20). One can now obtain the non-diagonal elements of the density operator, ρ, just by inverting the above linear equations. Besides, in case of a QTR for spatial qudits, the number of necessary slits at the plane-z is equal to the dimension D of the qudits. The positions of these slits can be determined by using the eigenvectors of the $D^2 - 1$ generators of $su(D)$ algebra [19].

A. Diagonal Elements

The measurement in the logical base, namely $\rho^{(z,z)}$, can be determined by coincidence measurements with the detectors just behind the double slits [13] or at the plane of image formation when lenses are placed in the path of the double-slits transmitted photons as showed in figure 1(b) [18]. In this last configuration, when the detector D_j is at position $x_{z,0} = -100\,\mu m$ ($x_{z,1} = +100\,\mu m$), it detects all photons that cross the slit $+$ $(-)$. By doing these measurements and normalizing the coincidences recorded for the four slits we obtained: $\rho_{++,++} = 0.028$, $\rho_{+-,+-} = 0.468$, $\rho_{-+,-+} = 0.462$ and $\rho_{--,--} = 0.042$.

B. Non diagonal Elements

The second type of coincidence measurements were done by positioning the signal detector at $x_{z,0} = -100$ μm or $x_{z,1} = +100\,\mu m$ and with the idler detector in the plane-z at the transversal positions $x_{y,0} = -\Delta/2$ ($-0.688mm$), $x_{y,1} = \Delta/2$ ($0.688mm$), $x_{x,0} = 0\,mm$ and $x_{x,1} = \Delta$ ($1.376\,mm$) (See figure 1(c)). When the idler detector is at the transverse position $x_{y,0}$ or $x_{y,1}$, the detector selects the idler photons in the $|f(x_{y,0})\rangle_i$ state or in the $|f(x_{y,1})\rangle_s$ state. When the idler detector is at the transverse position $x_{x,0}$ or $x_{x,1}$, it detects the idler photons in the $|f(x_{x,0})\rangle_i$ state or in the $|f(x_{x,1})\rangle_s$ state. With these eight measured coincidence numbers, we de-

termined the non diagonal elements of the density operator $\{\rho_{++,+-}, \rho_{+-,++}, \rho_{-+,--}, \rho_{--,-+}\}$. By repeating this detection procedure and reversing the roles of the signal and idler detectors (See figure 1(d)), we found the non diagonal elements $\{\rho_{++,-+}, \rho_{-+,++}, \rho_{+-,--}, \rho_{--,+-}\}$. We show below the explicit expressions that determine ρ_{++-+}

$$\mathrm{Re}(\rho_{++-+}) = \frac{\rho_{0+,0+}^{(x,z)} - \rho_{++++}\cos^2\theta_x - \rho_{-+-+}\sin^2\theta_x}{\sin 2\theta_x},$$

$$(22)$$

and

$$\mathrm{Im}(\rho_{++-+}) = \frac{-\rho_{0+,0+}^{(y,z)} + \rho_{++++}\cos^2\theta_y + \rho_{-+-+}\sin^2\theta_y}{\sin 2\theta_y},$$

$$(23)$$

where

$$\cos\theta_\mu = \frac{|r_+(x_{\mu,0})|}{\sqrt{|r_+(x_{\mu,0})|^2 + |r_-(x_{\mu,1})|^2}}.$$ (24)

The above expressions for $\mathrm{Re}(\rho_{++-+})$ and $\mathrm{Im}(\rho_{++-+})$ are obtained by inverting Eq. (21), with $L_s = 0, L_i = +$, both for $\lambda = x, \mu = z$ and for $\lambda = y, \mu = z$.

In the third measurement type shown in figure 1(e), signal and idler detectors are positioned in the detection plane-z at the positions $x_{\lambda,k}$ and $x_{\mu,l}$, with λ, μ being x or y and k,l being 0 or 1. This allows, by means of similar expressions to (22) and (23), the determination of $\{\rho_{++,--}, \rho_{--,++}, \rho_{+-,-+}, \rho_{-+,+-}\}$. This set of measurements correspond to local operations being applied to each of the down-converted photons, simultaneously.

C. The Reconstructed Density Operator

By performing the quantum tomographic reconstruction, as described above, we found the following form for the density operator in its matrix representation of our experiment

$$\rho = \begin{bmatrix} 0.028 & 0.083 + 0.004i & 0.081 + 0.005i & -0.129 + 0.062i \\ 0.083 - 0.004i & 0.468 & 0.444 - 0.058i & 0.097 - 0.008i \\ 0.081 - 0.005i & 0.444 + 0.058i & 0.462 & 0.096 - 0.006i \\ -0.129 - 0.062i & 0.097 + 0.008i & 0.096 + 0.006i & 0.042 \end{bmatrix}.$$ (25)

The elements of a density operator must satisfy the Schwarz inequality, i.e., $|\rho_{jk}| \leq \sqrt{\rho_{jj}\rho_{kk}}$, where $j,k =$ $++$, $+-$, $-+$ and $--$, if it really represents a quantum

state. This is not our case for the matrix element ρ_{++--}, since it can be seen that $|\rho_{++--}| > \sqrt{\rho_{++++}\rho_{----}}$. The reason for that are the experimental fluctuations present in the coincidence measurements which can affect the final result as we discussed in the Introduction. This discrepancy can be reduced by increasing the detection time. Even though our reconstructed density matrix presents properties which are not fully compatible with the quantum state description, it is possible to show that it is consistent with the theory developed in section II. This is done in the next section, where we also show experimental evidences of the good quality of our reconstruction.

IV. DISCUSSION AND CONCLUSION

The measured density operator shown in (25) can be approximately written as

$$\rho = 0.87|\Phi\rangle_{11}\langle\Phi| + 0.13|\Phi\rangle_{22}\langle\Phi|, \quad (26)$$

where the states $|\Phi\rangle$ are given by

$$
\begin{aligned}
|\Phi\rangle_1 &= 0.077e^{i\phi_1}|++\rangle + 0.704e^{i\phi_2}|+-\rangle \\
&\quad +0.699e^{i\phi_2}|-+\rangle + 0.099e^{i\phi_3}|--\rangle, \quad (27)
\end{aligned}
$$

and

$$
\begin{aligned}
|\Phi\rangle_2 &= 0.514|++\rangle + 0.502e^{i\theta}|+-\rangle \\
&\quad +0.501e^{i\theta}|-+\rangle + 0.483|--\rangle, \quad (28)
\end{aligned}
$$

with $\phi_1 \simeq \phi_2 \simeq \phi_3 \approx 4.2$ and $\theta = 0.07$.

However, the possibility to decompose the density operator, ρ, in terms of the projectors of a state, $|\Phi\rangle_1$, which has a high degree of entanglement and a state, $|\Phi\rangle_2$, that is of the form predicted by (5), is not sufficient for associating them with the states generated by each arm of the interferometer in our experiment. We still have to give an experimental evidence which corroborates with (26) as a reasonable approximation for the quantum state of the twin photons, i.e., we need to show that the values of $A = 0.87$ and $B = 0.13$, obtained mathematically, are reasonable for the probabilities of generating these states in each arm.

We measured the values of A and B by blocking one of the arms of the interferometer and detecting the transmitted coincident photons through the signal and idler double-slits. A (B) is the ratio between the coincidence rate when arm 2 (arm 1) is blocked and the total coincidence rate when both arms are unblocked. From this measurement we obtained, $A = 0.85 \pm 0.03$ and $B = 0.15 \pm 0.03$.

Another experimental evidence for the high value of A can be found in the fourth order interference pattern recorded (See figure 2). The interference pattern is recorded by using the configuration shown in figure 1(e).
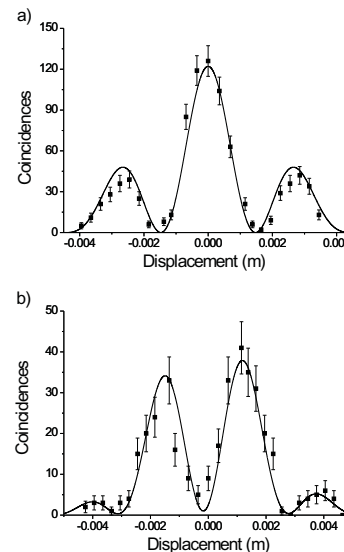

FIG. 2: Fourth order interference pattern as a function of D_s position. In (a), the detector idler was fixed at the transverse position $x = 0$ mm. In (b), it was fixed at the transverse position $x = 1376$ mm. The solid curves were obtained theoretically.

Fourth order interference pattern as a function of D_s position was recorded. In (a), the detector idler was fixed at the transverse position $x = 0$ mm. In (b), it was fixed at the transverse position $x = 1376$ mm. The solid curves were obtained theoretically [21], with A and B as free parameters. Since the state $|\Phi\rangle_1$ is almost a maximally entangled state we would expect to observe conditional interference patterns [21, 22] when both interferometer arms are unblocked. This would not be the case for high values of B. The conditionality can be clearly observed in our interference patterns. The reason for having the probability of generating the state from arm 1 much higher than the probability of generating the state of arm 2 is quite simple. The laser beam that cross arm 1 of the interferometer is focused at the double slit's plane-z_A, and the spatial correlation of the generated photons is such that it is more favorable to the transmission of the twin photon through the slits than it is when the photon pairs are generated by the pump beam that cross arm 2 [12]. These values can be properly manipulated by inserting attenuators at the interferometer.

These experimental observations confirm the good quality of the QTR performed on the two photon state and allow us to consider the states $|\Phi\rangle_1$ and $|\Phi\rangle_2$ as good approximations for the states generated by arm 1 and arm 2 of the interferometer used. Figure 3 shows a histogram of the real part of the matrix elements of (a) the measured density operator of (25), (b) the density operator given by (26) and, (c) the predicted density operator of section II. The agreement between the predicted and the measured density operator is good within the experimental errors. The largest error for the diagonal elements is only 3.5%. But, for the non-diagonal
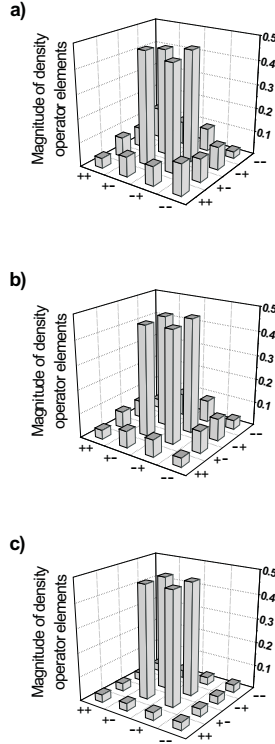
FIG. 3: Histogram of the real part of the matrix elements for (a) the measured density operator, (b) the density operator given by (26) and (c) the predicted density operator of Sec. II.

elements the propagated errors reaches 30% for their real parts and up to 65% for the imaginary parts. Again we remember that these errors can be decreased by increasing the detection time.

In conclusion, we have demonstrated that it is possible to generate a broad family of mixed states of spatial qudits by exploring the transverse correlation of the down-converted photons. A statistical mixture of spatial qubits were used to show the quantum tomographic reconstruction performed to measure its density operator. The process was discussed in details and experimental evidences for the good quality of the reconstruction performed were showed. Even though we had considered the state determination only for the case of qubits, it can be generalized and performed in a similar way for higher dimension systems in a mixed state. The importance of this work comes from the possibility of using spatial qudits for quantum communications protocols, where it requires the ability to characterize them in the presence both of noise source and of an undesired user at the quantum communication channel.

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